



# Logic for Computer Science. Knowledge Representation and Reasoning.

Lecture Notes  
for  
Computer Science Students  
Faculty EAIIB-IEiT AGH



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Other support material:

<http://home.agh.edu.pl/~ligeza>

<https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start>

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## Podstawowe zasady

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Serdecznie witam!

Dziękuję za wybór ISI!

Dziękuję za rejestrację na kurs i udział wykładach!

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1. Zapraszam na wykłady – obecność nie jest obowiązkowa – ale **wskazana – usilnie pożądana**,
2. Zapraszam do aktywnego udziału w wykładzie,
3. Pytania – tak!
4. Generalnie nie wyrażam zgody na nagrywanie (wyjaśnienie na wykładzie),
5. Materiały .pdf są udostępniane (przed lub po wykładzie).
6. Wykłady są dokumentowane na stronie  
<https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start>
7. Problemy – proszę zgłaszać, najlepiej z wyprzedzeniem, postaram się pomóc...

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## Logic: Let us start thinking...

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Where are we/you?

<https://www.youtube.com/watch?v=eBLJ7oixbOI>

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Logic: Direct correct proof:

<https://www.youtube.com/watch?v=YFZzLQN5qOU>

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Logic: Proof by contradiction

<https://www.youtube.com/watch?v=sRDwsfNDXak>

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Let us prove that  $2+2=5$

<https://www.youtube.com/watch?v=Cpug-FCZjhU>

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Let us prove that  $3=0$

<https://www.youtube.com/watch?v=SGUZ-8u10xM>

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## Basic Ideas behind this Course

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### Goals of the course:

- to teach selected methods of Logical Knowledge Representation,
- to teach selected methods of Logical Inference,
- with the focus on Propositional Logic Calculus, and also
- First Order Predicate Logic,
- with the ultimate sub-goals:
  - building logical models (KR),
  - performing logical reasoning (AR),
  - automated problem solving (APS),
  - analysis of logical properties (LPA).


$$\text{KR} + \text{AR} \longrightarrow \text{APS} + \text{LPA}$$


- to keep the course at understandable basic level and practical applications rather than just theory:
  - necessary background knowledge — but also in an informal way,
  - modern tools — if available,
  - examples + applications,
  - further references; internet sources.
- a reference point: CS157: <http://logic.stanford.edu/classes/cs157/current/>

## General Principles

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**Warning:** This presented course support material is not error-free. Everybody using these texts must do it with full comprehension and on his/her own responsibility.

!!!  !!! – identifies (some) pages where **fake info** is presented **on purpose**. Further explanation – during the lecture. If you are not present on the lecture – try to identify the fake info – and further think, why it was put there...

!?!  !?! – identifies (some) pages where a trick is hidden or a tricky problem is asked to be solved; be aware that you should locate the problem and know how to solve it! Again, if not present on the lecture – do it by yourself...

1. **Goal:** to learn some knowledge and practical skills in **logic**.
2. **Syllabus:** see <https://syllabusy.agh.edu.pl/pl/wizard/token/step-1/26b82c67-338a-4321-b075-fe1edb3a1e80?apiKey=5de78a88decf9>
3. **Formal frames:** Regulamin Studiów w AGH<sup>1</sup>, but also **logic** and **reasonable, common sense approach**.
4. **Example of particular practical rules GEIST:** <http://geist.agh.edu.pl/pub:teaching:gris>
5. **Forms of work and knowledge acquisition:**
  - lecture,
  - blackboard exercises,
  - *e-learning* (Wikipedia, Coursera,...),

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<sup>1</sup><http://http://www.agh.edu.pl/pracownicy/dokumenty/regulaminy/>

- personal, independent study,
  - discussion, questions and possible answers,
  - consultations.
6. Participation in practical classes is obligatory.
  7. Personal attendance and *mindfulness* (*uwaga*) at/during the lectures is **strongly recommended**.
  8. A large dose of Enthusiasm, additional activity – supported by personal skills are welcome.
  9. **It is strongly recommended** to take personal notes.
  10. Everybody builds her/his own **Logical Knowledge Base!**
  11. **Full understanding of the material** is of principal importance!.
  12. You can ask questions – any time and *almost* on any topics.
  13. Tests, exams, practical exercises, etc. — it is obligatory that you present **original work on your own**.
  14. The general principles — according to **Regulamin Studiów**.
  15. **We have only 10 regular lectures...**
  16. A schedule/report of lectures: [https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start#logika\\_dla\\_informatykow2020\\_2021](https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start#logika_dla_informatykow2020_2021)
  17. **Only 28 hours...No final exam???**

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## Bibliography

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1. **Mordechai Ben-Ari: Mathematical Logic for Computer Science (Logika matematyczna w informatyce). Springer-Verlag, London, 2012 (WN-T, Warszawa, 2005, 2006).**
2. Kenneth A. Ross i Charles R. B. Wright: Discrete Mathematics (Matematyka dyskretna). WN PWN, 2013.
3. Antoni Ligęza: Logical Foundations for Rule-Based Systems. Springer-Verlag, Berlin, 2006. Wydawnictwo AGH, Kraków, 2005.
4. Michael R. Genesereth, Nils J. Nilsson: Logical Foundations of Artificial Intelligence. Morgan Kaufmann Publishers, Inc., Los Altos, California, 1987.
5. Zbigniew Huzar: Elementy logiki dla informatyków. Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław, 2007.
6. Stuart Russell, Peter Norvig: Artificial Intelligence. A Modern Approach. Pearson, 2010.
7. Marek Wójcik: Zasada rezolucji. Metoda automatycznego wnioskowania. PWN, Warszawa, 1991.
8. C. L. Chang and R. C. T. Lee: Symbolic Logic and Mechanical Theorem Proving. Academic Press, 1973.
9. Ronald J. Brachman and Hector J. Levesque: Knowledge Representation and Reasoning. Morgan Kaufmann, 2004.
10. Frank van Harmelen, Vladimir Lifschitz, Bruce Porter (Eds.): *Handbook of Knowledge Representation*. Elsevier B.V., Amsterdam, 2008.  
<http://ii.fmph.uniba.sk/~sefranek/kri/handbook/>

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## Support Material: On the Net

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### Mathematical Logic for Computer Science:

<https://www.weizmann.ac.il/sci-tea/benari/research-activities/mathematical-logic-computer-science-third>

### Stanford on-line Course:

<https://www.coursera.org/learn/logic-introduction>

### Lectures - On-Line Documentation and Archives:

<https://ai.ia.agh.edu.pl/pl:dydaktyka:logic:start>

1. Wikipedia-pl: [http://pl.wikipedia.org/wiki/Logika\\_matematyczna](http://pl.wikipedia.org/wiki/Logika_matematyczna)
2. Wikipedia-en: <http://en.wikipedia.org/wiki/Logic>
3. AI-Lab-Prolog: [http://ai.ia.agh.edu.pl/wiki/pl:prolog:prolog\\_lab](http://ai.ia.agh.edu.pl/wiki/pl:prolog:prolog_lab)
4. EIS-KRR: <http://ai.ia.agh.edu.pl/wiki/pl:dydaktyka:krr:start>
5. ALI-home: [home.agh.edu.pl/~ligeza](http://home.agh.edu.pl/~ligeza)
6. David Poole and Allen Mackworth: Artificial Intelligence. Foundations of Computational Agents. <http://artint.info/>
7. Ulf Nilsson and Jan Maluszynski: Logic, Programming and Prolog. <http://www.ida.liu.se/~ulfni/lpp/>



## Logics - what is that?

**Definition 1** *Logika (logos — rozum, słowo, myśl) — nauka o sposobach jasnego i ścisłego formułowania myśli, o regułach poprawnego rozumowania i uzasadniania twierdzeń.*

**Definition 2** *Logic (from the Ancient Greek ) is the systematic study of the forms of inference, the relations that lead to the acceptance of one proposition, the conclusion, on the basis of a set of other propositions, the premises. More broadly, logic is the analysis and appraisal of arguments.*

**Logic** = (Axioms) + (Formal Models) + (Valid Inference Rules)

### Tools:

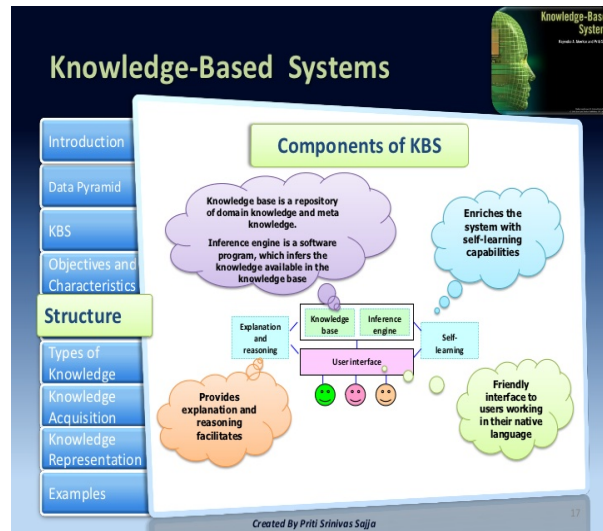
- Formal Models – built with Formal Languages:
  - alphabet,
  - syntax,
  - semantics,
  - axioms,
  - inference rules,
  - inference strategies (methods of Theorem Proving);

## Supplementary Tools

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- Visualization — e.g. Venn's Diagrams,
- Tables (complete specification of cases),
- Trees (better visual presentation),
- Diagrams (graphs; AND-OR graphs; schemes),
- Software tools (e.g. SAT solvers, Prolog, ASP, ATP).

## Knowledge-Based Systems: Basic Concepts



We need some tools:

- A formal language for KRR:
  - alphabet,
  - syntax,
  - semantics,
  - inference rules;
- various goals — various types of reasoning,
- inference strategies,
- knowledge acquisition,
- knowledge verification,
- minimal knowledge representation (uniform representation),
- internal knowledge structure (knowledge graphs),
- user interface, explanations, learning, adaptation, optimization.

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## Przykłady... wieloznaczność języka naturalnego

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Żona do męża informatyka

- Kup parówki, a jak będą jajka -- to kup 10.

Mąż w sklepie:

- Są jajka?

- Są.

- To poproszę 10 parówek.

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Przykłady wyrażen w języku naturalnym:

- *Mądrej głowie dość po słowie* (Wise head - a word is enough).
  - *Mądrej głowy włos się nie trzyma.*
  - *Dobry kogut to chudy kogut* (A good cock is a skinny cock).
  - *Iloma językami władasz, tylekroć jesteś człowiekiem*  
(How many languages do you speak, so many times you are human;  
"Wie viele Sprachen du sprichst, sooft mal bist du Mensch.").
- Logika to także język.
- *Historia uczy jednego: nigdy, nikogo, niczego nie nauczyła.*  
Czy to zdanie może być prawdziwe?

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## Logic – a tool for Knowledge Representation and Reasoning (KRR)

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### Observation:

Not all the systems/processes/... can be modelled with **numerical tools**.

Simultaneously, Natural Language (NL):

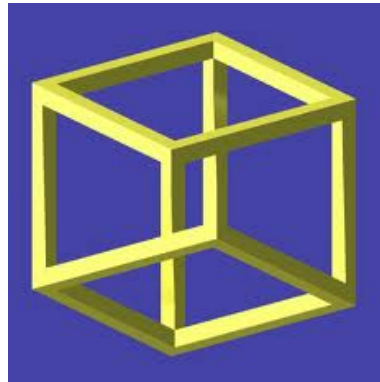
- is often inadequate/too poor/....,
- imprecise, incomplete,
- ambiguous– there are different interpretations possible,
- hard for automated processing,
- can lead to **inconsistent models/theories**,
- is redundant,
- requires the knowledge of *domain ontology*.

Formal language – selected requirements:

- adequate,
- precise,
- unambiguous,
- automatic processing,
- should assure (**consistency**),
- should allow to produce **validit/sound** conclusions,
- can assure **completeness**,
- should be not redundant.

## Something has gone wrong...

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## What one needs logic for?

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Po co komu krawat? Sprzedawca krawatów...

- logic allows to formulate definitions of concepts (*marriage, fast tramway, średnioroczny wzrost cen, jakość wykształcenia, jakość kształcenia, jakość procesu kształcenia,...*),
- and the relationship among them — np. taxonomies, mutual relations/roles, ontologies,
- logic introduces order into discussion; rational discussion based on *argumentation*,
- logic provides formal **Knowledge Representation**,
- logic provides valid/correct **inference methods** — deduction,
- but also: **induction** and **abduction**,
- logically specified knowledge can undergo formal analysis:
  - internal; consistency (wewnętrzna niesprzeczność/spójność),
  - completeness (zupełność),
  - minimal representation (minimalna reprezentacja),
  - logical consequence (wynikanie logiczne — logiczna konsekwencja),
  - satisfiability or unsatisfiability (spełnialność lub niespełnialność).
- logic is a formal tool (is widely applied in various domains...).

Where logic is not applied/applicable?? What does not undergo logical rules of reasoning?

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## Some hot areas of application

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- Computer Programming,
- Knowledge-Based Systems (KBS),
- Automated Theorem Proving (ATP),
- Logic Programming,
- SAT,
- Constraint Programming,
- Answer Set Programming; ASP,
- Systems Design and Analysis,
- Model Checking,
- systems/software certification.



## Logic – OK – but still some problems...

### Natural language:

#### Paradoks kłamcy:

- *I always lie!* (Eubulides),
- *The Cretans always lie* (Epimenides; he was a Cretan himself),
- Page 1: *The sentence on the other side is true*; Page 2: *The sentence on the other side is false.*
- **Mukator** — Co to jest mukator? To jest coś, co *nie daje się zdefiniować*.

### Formal language:

#### Matematyka:

- Paradox of the set of all sets (Cantor, 1899),
- Russel's paradox (1901): consider  $V = \{X : X \notin X\}$ . Does it hold that:  $V \in V$ ?
- Barber Paradox: *A barber shaves all residents of the city who do not shave themselves; what should he do with himself?*

**Examples of paradoxes:** <http://pl.wikipedia.org/wiki/Paradoks>;  
<https://en.wikipedia.org/wiki/Paradox>

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## Formal Knowledge Representation Language: Specification of Requirements

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### Knowledge Representation Language:

- unique interpretation,
  - expressive power and precision, but
  - automated processing,
  - adequate for the domain,
  - readable for man and machine,
  - extensible,
  - ...
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### Knowledge Base:

- **consistent**, (internal consistency)
- **complete**,
- **valid; sound**, (external consistency)
- **non-redundant**,
- **efficient in problem solving**, (optimal)
- ...

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## Logic — how it works?

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### Logic – formal language:

- alphabet,
- *syntax*,
- *semantics*,
- axiomatization

$$\models p \vee \neg p$$

$$\not\models p \wedge \neg p$$

- equivalency-preserving transformations,
- inference rules (logical consequence),
- inference chain – derivation,
- problem, hypothesis, question,
- answer, solution, proof.

### System modeling :

- language selection – for adequate modeling,
- model building (inputs – outputs – internal structure),
- model analysis (verification, validation),
- model exploration – theorem proving, SAT,
- model tuning and adaptation; learning.

## Example: a simple intro to Propositional Calculus

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### Alphabet:

- $P$  — set of propositional symbols,
- logical connectives:  $\neg, \wedge, \vee, \Rightarrow$ ,
- parentheses.

### Syntax:

- every  $p \in P$  is a formula (atomic formula; positive literal),
- if  $\phi, \psi$  are formulas, then:
  - $\neg(\phi)$  is a formula (also:  $\neg(\psi)$ ),
  - $(\phi) \wedge (\psi)$  is a formula,
  - $(\phi) \vee (\psi)$  is a formula,
  - $(\phi) \Rightarrow (\psi)$  is a formula;
  - *no other expression is a formula.*

Each well-formed formula (wff) has a uniquely defined [tree structure](#).

### Semantics:

Interpretation — any function  $I$  of the form:

$$I: P \rightarrow \{\mathbf{T}, \mathbf{F}\}$$

The definition of interpretation is [extended](#) over the set of all formulae (how to do that ???).

Notation:  $I(\phi) = \mathbf{T}$  we shall write  $\models_I \phi$ ;  $I(\phi) = \mathbf{F}$  we shall write  $\not\models_I \phi$ .

For any wff a [truth table](#) can be built.

The symbols  $\mathbf{T}$  (*True, true*) and  $\mathbf{F}$  (*False, false*) are often replaced with **1** and **0**, respectively.

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## Rules of inference

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[https://en.wikipedia.org/wiki/List\\_of\\_rules\\_of\\_inference](https://en.wikipedia.org/wiki/List_of_rules_of_inference): examples + example derivations

Example rules of inference (derivation rules):

**Detachment Rule (*Modus Ponens*; Reguła odrywania):**

$$\frac{\phi, \phi \Rightarrow \psi}{\psi}$$

**Contraposition (*Modus Tollens*; Wnioskowanie przez zaprzeczenie):**

$$\frac{\phi \Rightarrow \psi, \neg\psi}{\neg\phi}$$

**Disjunctive Syllogism; Disjunction Elimination by Contradiction (Reguła potwierdzania przez zaprzeczenie (wykluczenie)):**

$$\frac{\neg\phi, \phi \vee \psi}{\psi}$$

**Transitivity Rule (Reguła przechodności):**

$$\frac{\phi \Rightarrow \varphi, \varphi \Rightarrow \psi}{\phi \Rightarrow \psi}$$

**Resolution Rule (Reguła rezolucji):**

$$\frac{\phi \vee \neg p, p \vee \psi}{\phi \vee \psi}$$

**What about the following rules? !!!  !!!**

**Implication + positive conclusion?**

$$\frac{\phi \Rightarrow \psi, \psi}{\phi}$$

**Implication + negative premise**

$$\frac{\neg\phi, \phi \Rightarrow \psi}{\neg\psi}$$

**2 Implications + conjunction**

$$\frac{p \wedge q, p \Rightarrow r, q \Rightarrow s}{r \wedge s}$$

**Proof by Cases: 2 Implications + disjunction**

$$\frac{p \vee q, p \Rightarrow r, q \Rightarrow r}{r}$$

**Constructive Dilemma: 2 Implications + disjunction**

$$\frac{p \vee q, p \Rightarrow r, q \Rightarrow s}{r \vee s}$$

**Dual Resolution Rule (Reguła rezolucji dualnej):**

$$\frac{\phi \wedge \neg p, p \wedge \psi}{\phi \wedge \psi}$$

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## Example — układ EX-OR

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```
% Definicje działania bramek podstawowych
not(1,0).
not(0,1).

and(0,0,0).
and(0,1,0).
and(1,0,0).
and(1,1,1).

or(0,0,0).
or(0,1,1).
or(1,0,1).
or(1,1,1).

% Definicja przykładowego układu - xor

xor(Input1,Input2,Output) :-
not(Input1,N1),
not(Input2,N2),
and(Input1,N2,N3),
and(Input2,N1,N4),
or(N3,N4,Output).
```

### An Example — układ EX-OR

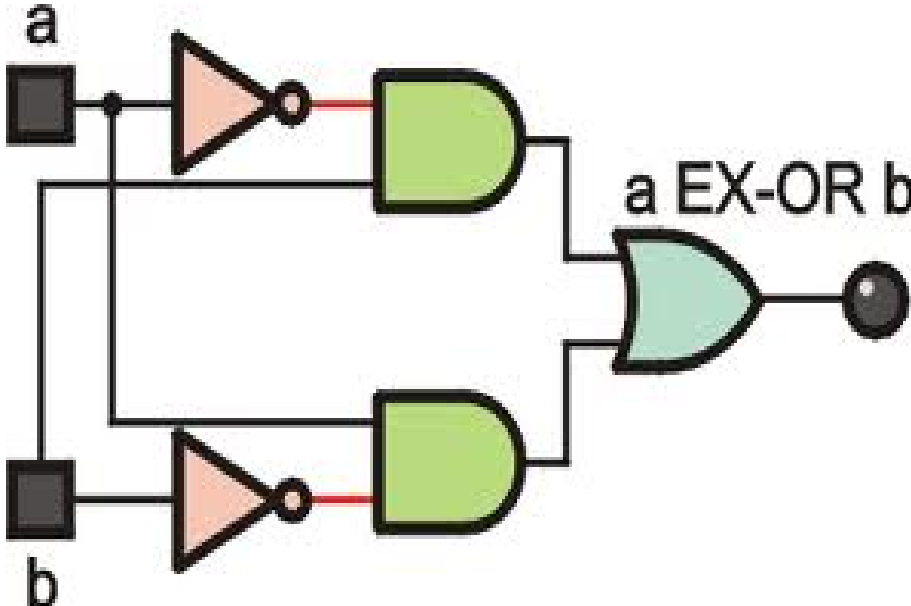
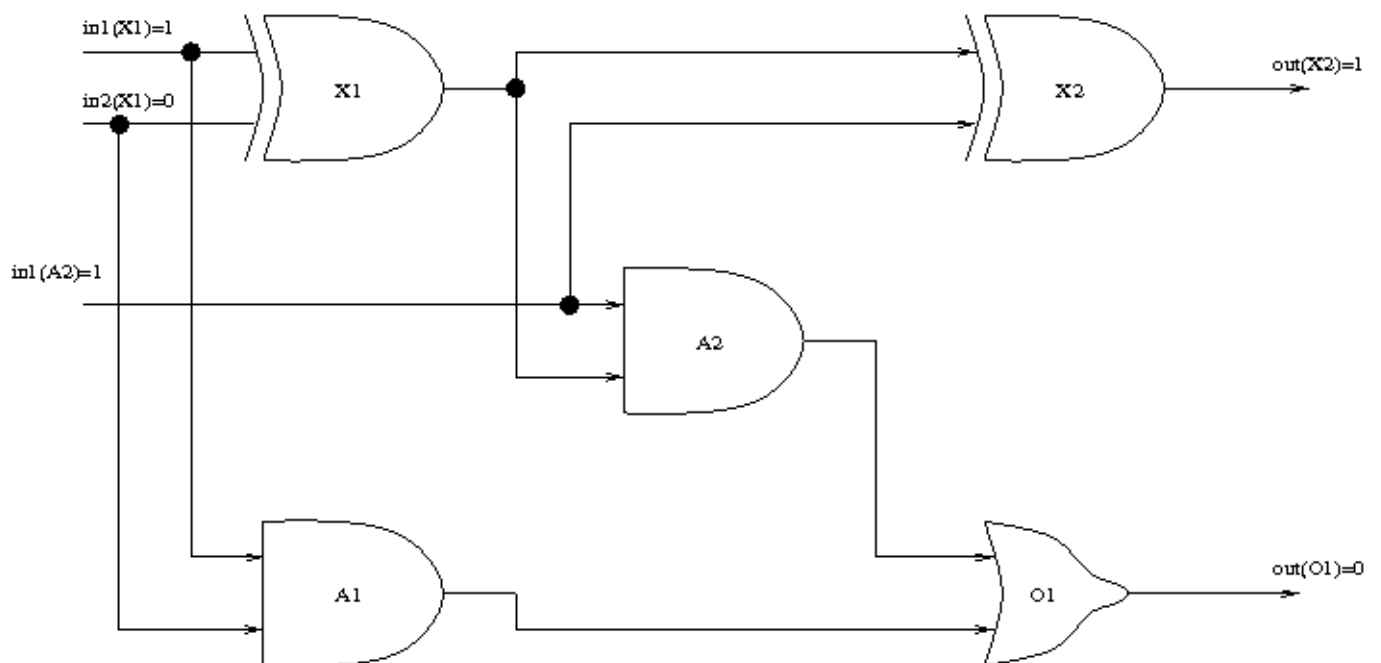


Figure 1: EX-OR digital circuit



## A Practical Example

```
foo(I1, I2, I3, O1, O2) :-  
  xor(I1, I2, X1),  
  xor(X1, I3, O1),  
  and(I1, I2, A1),  
  and(X1, I3, A2),  
  or(A1, A2, O2).
```



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## Logical inferences — examples

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### Examples of logical inference:

- **deduction** — fundamental logical inference (hypotheses proving);
- SAT: satisfiability checking — model search, constraint analysis;
- detection of inconsistencies; consistency-based reasoning (CBR);
- **tautology** verification;
- **abduction** — making hypotheses (finding out the causes);
- **induction** — generalization;
- case-based reasoning, reasoning by analogy, plausible reasoning; fuzzy reasoning, probabilistic reasoning,...

### Examples – applications – and their classification:

- deductive theorem proving (deduction rules, Herbrand, Fitch, forward checking),
- formula for the sum of the infinite geometric sequence (deduction vs. induction),
- SEND + MORE = MONEY,
- the Einstein/Zebra problem,
- diagnosis of multiplier-adder,
- a promise of earnings above average for everyone,
- rational result of power of two irrational numbers,
- polynomial example ( $P > 0$  and  $\forall \epsilon > 0: P < \epsilon$ ),

- practical examples:
  - 7-segment display - for 16-bar code
  - 3/5 or 4/7 voting system
  - lock-2-doors to the bank,
  - a 2-way pass-through switch,
  - railway crossing,

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## Logic for KRR – Tasks and Tools

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- Theorem Proving – Verification of Logical Consequence:

$$\Delta \models H;$$

- Automated Inference – Derivation:

$$\Delta \vdash H;$$

- SAT (checking for models) – satisfiability:

$$\models_I H;$$

- un-SAT verification – unsatisfiability:

$$\not\models_I H \quad \text{for any interpretation } I;$$

- Tautology verification (completeness):

$$\models H$$

- valid inference rules – checking:

$$(\Delta \vdash H) \longrightarrow (\Delta \models H)$$

- complete inference rules – checking:

$$(\Delta \models H) \longrightarrow (\Delta \vdash H)$$

## Inference example

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**A** – signal from process,

**P** – signal added to a queue,

**B** – signal blocked by process,

**D** – signal received by process,

**S** – state of the process saved,

**M** – signal mask read,

**H** – signal management procedure activated,

**N** – procedure executed in normal mode,

**R** – process restart from context,

**I** – process must re-create context.

Rules — axiomatization:

$A \longrightarrow P,$

$P \wedge \neg B \longrightarrow D,$

$D \longrightarrow S \wedge M \wedge H,$

$H \wedge N \longrightarrow R,$

$H \wedge \neg R \longrightarrow I,$

Facts:

$A, \neg B, \neg R.$  What can be inferred from the facts? What are the logical consequences of the KB? Which facts are True and which ones are False?

Is the system complete?

## A Problem to Consider: Tracking the Murderer

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Some knowledge specification — in natural language:

- If Sarah was drunk then either James is the murderer or Sarah lies,
- Either James is the murderer or Sarah was not drunk and the crime took place after midnight,
- If the crime took place after midnight then either James is the murderer or Sarah lies,
- Sarah does not lie when sober.

Introduction symbols and transformation to formal specification:

- $A$  = James is the murderer,
- $B$  = Sarah is drunk,
- $C$  = Sarah lies,
- $D$  = The murder took place after midnight.

$$B \implies A \vee C$$

$$A \vee (\neg B \wedge D)$$

$$D \implies (A \vee C)$$

$$C \implies B$$

Questions:

Who is the murderer? Which facts are true/false? Is the system consistent? How many models does it have (if consistent)? What are the exact models?

## Example: Unicorn

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Given the following Knowledge Base (KB):

- If the unicorn is mythical, then it is immortal
- If the unicorn is not mythical, then it is a mortal mammal
- If the unicorn is either immortal or a mammal, then it is horned
- The unicorn is magical if it is horned

answer the following questions:

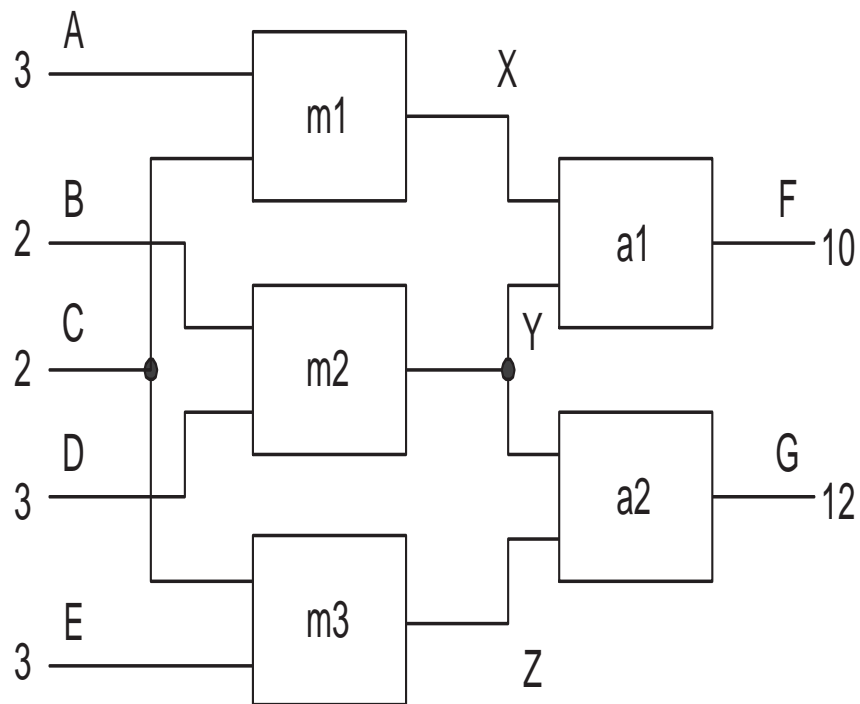
- Is the unicorn mythical? ( $M$ )
- Is it magical? ( $G$ )
- Is it horned? ( $H$ )

In terms of logic:

$$\text{KB} \models G, H, M$$

$$\text{KB} \vdash G, H, M$$

## Abductive Inference and Consistency-Based Reasoning



Questions:

- Does the system work OK (Fault Detection)?
- Which component(s) is(are) faulty (Fault Isolation)?
- What are the diagnoses (minimal diagnoses)?